John R. Hicks Value and Capital (1939)

Chapter I Utility and Preference

1. THE pure theory of consumer's demand, which occupied a good deal of the attention of Marshall and his contemporaries, has received far less notice in the present century. The third book of Marshall's *Principles* still remains the last word on the subject so far as books written in English are concerned. Now Marshall's theory of demand is no doubt admirable,* but it is remarkable that it has remained so long upon such an unquestioned eminence. This would be explicable if there were really no more to say on the subject, and if every step in Marshall's analysis were beyond dispute. But this is clearly not the case; several writers have felt very uncomfortable about Marshall's treatment,** and it is actually the first step, on which everything else depends, which is the most dubious.

*My own experience has been that further investigation has only increased my admiration for Marshall's theory; I hope the reader will find the same. ** For example, Wicksteed, *Common Sense of Political Economy*, chs. i-3; Robbins, *Nature and Significance of Economic Science*, ch. 6.

Let us first remind ourselves of the bare outline of Marshall's main argument.* A consumer with a given money income is confronted with a market for consumption goods, on which the prices of those goods are already determined; the question is, How will he divide his expenditure among the different goods? It is supposed, for convenience, that the goods are available in very small units.** It is assumed that the consumer derives from the goods he purchases so much 'utility', the amount of utility being a function of the quantities of goods acquired; and that he will spend his income in such a way as to bring in the maximum possible amount of utility. But utility will be maximized when the marginal unit of expenditure in each direction brings in the same increment of utility. For, if this is so, a transference of expenditure from one direction to another will involve a greater loss of utility in the direction where expenditure is reduced than will be compensated by the gain in utility in the direction where expenditure is increased (from the principle of diminishing marginal utility). Total utility must therefore be diminished, whatever transfer is made. Since, with small units, the differences between the marginal utilities of two successive units of a commodity may be neglected, we can express the conclusion in another way: the marginal utilities of the various commodities bought must be proportional to their prices

* Principles, iii. 5. 2.

** This convenient assumption of continuity does, of course, always falsify the situation a little (or sometimes more than a little) as far as the individual consumer is concerned. But if our study of the individual consumer is only a step towards the study of a group of consumers on the market, these falsifications can be trusted to disappear when the individual demands are aggregated.

Marshall's argument therefore proceeds from the notion of maximizing total utility, by way of the law of diminishing marginal utility, to the conclusion that the marginal utilities of commodities bought must be proportional to their prices.

But now what is this 'utility' which the consumer maximizes? And what is the exact basis for the law of diminishing marginal utility? Marshall leaves one

Space for Notes

uncomfortable on these subjects. However, further light on them was thrown by Pareto.

2. Pareto's *Manuel d'économie politique (1909)* has to be reckoned as the other classical treatment of the theory of consumer's demand, from which any modern investigation must begin. It is not that Pareto's book, as a whole, is at all comparable with Marshall's. The *Manuel* purports to be a sort of general *Principles;* but most problems are treated by it quite superficially, while its famous theory of General Equilibrium is nothing else but a more elegant restatement of the doctrines of Walras. However, on this particular matter of utility theory Pareto was a specialist, and his investigations well deserve attention. Since they are not very familiar to English readers, I shall summarize the relevant arguments rather carefully.

Pareto started off, originally, from the same utility theory as Marshall; the argument we have just summarized would have been quite acceptable to him also in the first stage of the development of his ideas. But instead of proceeding afterwards, as Marshall did, to concentrate attention upon the demand for a single commodity (and thus to investigate the relation between the curve of diminishing marginal utility and the demand curve), Pareto turned his attention to the problem of related - complementary and competitive-- goods. Here he made an extension of the earlier analysis; or rather, something which started as an extension but ended as a revolution.

For the purpose of studying related goods, Pareto took over from Edgeworth,* a geometrical device --the Indifference Curve. When we are concerned, like Marshall, with one commodity only, we can draw a total utility curve, measuring amounts of that commodity along one axis, and total amounts of utility derived from those various amounts of commodity along the other axis. Just in the same way, when we are interested in two commodities, we can draw a utility surface. Measuring quantities of the two commodities X and Y along





utility derived from that particular collection of quantities. Joining the tops of these ordinates, we get a 'utility surface' (Fig. I overleaf).

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* Mathematical Psychics, pp. 21-2.
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In principle, this is simple enough; but three-dimensional diagrams are awkward things to handle. Fortunately, having once visited the third dimension, we need not stay there. The third dimension can be eliminated, and we can return to two.

Instead of using a three-dimensional model, we can use a map (Fig. 2). Keeping quantities of the two commodities X and Y along the two axes, we can mark off on the horizontal diagram the contour lines of the utility surface (the broken line in Fig. 1). These are the indifference curves. They join all those points which correspond to the same height in the third dimension, that is, to the same total utility. If P and P are on the same indifference curve, that means that the total utility derived from having *PM and* PN is the same as that derived from having P'M' and P'N'. If P'' is on a higher indifference curve than P (the curves will have to be numbered so as to distinguish higher from lower), then P''M'' and P''N''' will give a higher total utility than *PM and PN*.



What will be the shape of these indifference curves? So long as each commodity has a positive marginal utility, the indifference curves must slope downwards to the right. For if X has a positive marginal utility, an increase in the quantity of X, unaccompanied by any change in the quantity of Y (that is to say, a simple movement to the right on the diagram), must increase total utility, and so bring us on to a higher indifference curve. Similarly, a simple movement upwards must lead on to a higher indifference curve. It is only possible to stay on the same indifference curve if these movements are compensated --X increased and Y diminished, or X diminished and Y increased. The curves must therefore slope downwards to the right.

The slope of the curve passing through any point P has indeed a very definite and important meaning. It is the amount of Y which is needed by the individual in order to compensate him for the loss of a small unit of X. Now the gain in utility got by gaining such an amount of Y equals amount of Y gained times marginal utility of Y; the loss in utility got from losing the corresponding amount of X equals amount

of X lost times marginal utility of X (so long as the quantities are small). Therefore, since the gain equals the loss, the slope of the curve

- = (amount of Y gained)/(amount of X lost)
- = (marginal utility of X)/(marginal utility of Y)

The slope of the curve passing through P measures the ratio of the marginal utility of X to the marginal utility of Y, when the individual has quantities PM and PN of X and Y respectively.

Have we any further information about the shapes of the curves? There ought, it would seem, to be some way of translating into terms of this diagram –the principle of diminishing marginal utility. At first sight, it looks as if such a translation were possible. As one moves along an indifference curve one gets more X and less Y. The increase in X diminishes the marginal utility of X, the diminution in Y increases the marginal utility of Y. On both grounds, therefore, the slope of the curve must diminish. Falling curves, whose slope diminishes as we move to the right, will be convex to the origin, as they have been drawn in the diagram.

But does this quite necessarily follow? As far as the direct effects just taken into account are concerned, it must; but there are other indirect effects to take into account too. The increase in X may affect not only the marginal utility of X, it may also affect the marginal utility of Y. With such related goods the above argument does not necessarily follow. Suppose that the increase in X lowers the marginal utility of Y, and the diminution in Y raises the marginal utility of X; and that these cross-effects are considerable. Then the cross-effects may actually offset the direct effects, and a movement along the indifference curve to the right_may actually increase the slope of the curve. This is no doubt a very queer case, but it is consistent with diminishing marginal utility. Diminishing marginal utility and convexity of the indifference curves are not the same thing.

3. We come now to the really remarkable thing about indifference curves the discovery which shunted Pareto's theory on to a different line from Marshall's, and opened a way to new results of wide economic significance.

Suppose that we have a consumer with a given money income, who is spending the whole of that income upon the two commodities X and Y, no others entering into the picture. Suppose that the prices of those commodities are given on the market. Then we can read off the amounts that he will buy directly from his indifference map, without any information about the amounts of utility he derives from the goods.

Mark off a length OL along the X-axis (Fig. 3), representing the amount of X which he could buy if he spent all of his income upon X; and an amount OM on the Y-axis, representing the amount of Y he could buy if he spent all his income upon Y; and join *LM*. Then any point on the line *LM* represents a pair of quantities of the two commodities which he could buy out of his income. Starting from *L*, in order to acquire some Y, he will have to give up X in the proportion indicated by the ratio of their prices; and the price-ratio is indicated by the slope of the line *LM*.



Through any point on the line LM there will pass an indifference curve; but usually the line LM will intersect the indifference curve. If this happens the point cannot be one of equilibrium. For, by moving along the line LM in one direction or the other, the consumer will always be able to get on to a higher indifference curve, which gives him greater utility. He is therefore not maximizing his utility at that particular point.

It is only when the line *LM* touches an indifference curve that utility will be maximized. For at a point of tangency, the consumer will get on to a lower indifference curve if he moves in either direction.

Tangency between the price-line and an indifference curve is the expression, in terms of indifference curves, of the proportionality between marginal utilities and prices.

4. Thus we can translate the marginal utility theory into terms of indifference curves; but, having done that, we have accomplished something more remarkable than a mere translation. For, in the process of translation, we have left behind some of the original data; and yet we have arrived at the desired result all the same.

In order to determine the quantities of goods which an individual will buy at given prices, Marshall's theory implies that we must know his utility surface; Pareto's theory only assumes that we must know his indifference map. And that conveys less information than the utility surface. It only tells us that the individual prefers one particular collection of goods to another particular collection; it does not tell us, as the utility surface purports to do, how much the first collection is preferred to the second.

The numbers which we give to the indifference curves are indeed wholly arbitrary; it will be convenient for them to rise as we go on to higher curves, but the numbers can be 1, 2, 3, 4..., 1, 2, 4, 7..., 1, 2, 7, 10..., or any ascending series we like to take.

Pareto's little piece of geometry thus resulted in a conclusion of wide methodological importance. It is necessary, in any theory of value, to be able to define just what we mean by a consumer of 'given wants' or 'given tastes'. In Marshall's theory like that of Jevons and Walras, and the Austrians) 'given wants' is interpreted as meaning a given utility function, a given intensity of desire for any particular collection of goods. This assumption has made many people uncomfortable, and it appears from Pareto's work that it is not a necessary assumption at all. 'Given wants' can be quite adequately defined as a given *scale of*

preference need only suppose that the consumer has a preference for one collection of goods rather than another, not that there is ever any sense in saying that he desires the one collection 5 per cent. more than the other, or anything like that.

Now of course this does not mean that if any one has any other ground for supposing that there exists some suitable quantitative measure of utility, or satisfaction, or desiredness, there is anything in the above argument to set against it. If one is a utilitarian in philosophy, one has a perfect right to be a utilitarian in one's economics. But if one is not (and few people are utilitarians nowadays), one also has the right to an economics free of utilitarian assumptions.

From this point of view, Pareto's discovery only opens a door which we can enter or not as we feel inclined. But from the technical economic point of view there are strong reasons for supposing that we ought to enter it. The quantitative concept of utility is not necessary in order to explain market phenomena. Therefore, on the principle of Occam's razor, it is better to do without it. For it is not, in practice, a matter of indifference if a theory contains unnecessary entities. Such entities are irrelevant to the problem in hand, and their presence is likely to obscure the vision. How important this is can only be shown by experience; I shall hope to convince the reader that it is of some considerable importance in this case.

5. Acting on this principle, we have now to inquire whether a full theory of consumer's demand at least as thoroughgoing as Marshall's cannot be built up from the assumption of a scale of preference In constructing such a theory it will be necessary every time to reject any concept which is at all dependent upon quantitative utility, so that it cannot be derived from the indifference map alone. We start off from the indifference map alone; nothing more can be allowed.

In undertaking this reconsideration we lose the help of Pareto; for even after Pareto had established his great proposition, he continued to use concepts derived from the earlier set of ideas. The reason was, perhaps, that he did not take the trouble to rework his earlier conclusions in the light of a proposition which he only reached at a rather late stage of his work in economics.* However that may be, he missed an opportunity.

*Further, much of the energy which he had left for the subject was expended upon chasing a will-o'-the-wisp. When more than two goods are being consumed, it is possible that the differential equation of the preference system may not be integrable. This point fascinates mathematicians, but it does not seem to have any economic importance at all, the only problems to which it could conceivably be relevant being much better treated by other methods. Cf. Pareto, *Manuel, pp. 546-57 ; 'Economie* mathematique' (in *Encyclopidie del Sciences mathématiques* 1911), *pp. 597, 614*. A recent discussion of nonintegrability will be found in N. Georgescu-Roegen, 'The Pure Theory of Consumer's Behaviour' (*Q.J.E., Aug. 1936*).

The first person to take the opportunity was the Russian economist and statistician Slutsky, in an article published in the Italian *Giornale degli Economisti* in 1915.* The theory to be set out in this chapter and the two following is essentially Slutsky's; although the exposition is modified by the fact that I never saw Slutsky's work until my own was very far advanced, and some time after the substance of these chapters had been published in *Economica* by R. G. D. Allen and myself.** Slutsky's work is highly mathematical, and he does not give much discussion about the significance of his theory. These things (and the date of its publication) perhaps explain why it remained for so long without influence, and had to be rediscovered. The present volume is the first systematic exploration of the territory which Slutsky opened up.

* E. Slutsky, 'Sulla teoria del bilancio del consumatore' (*G.d.E.*, July 1915). See also R. G. D. Allen, 'Professor Slutsky's Theory of Consumer's Choice' (*Review of Economic Studies*,

1936).

** 'A Reconsideration of the Theory of Value' (Economics, 1934).

6. We have now to undertake a purge, rejecting all concepts which are tainted by quantitative utility, and replacing them, so far as they need to be replaced, by concepts which have no such implication.

The first victim must evidently be marginal utility itself. If total utility is arbitrary, so is marginal utility. But we can still give a precise meaning to the ratio of two marginal utilities, when the quantities possessed of both commodities are given.* For this quantity is represented by the slope of an indifference curve; and that is independent of the arbitrariness in question.

* On the other hand there will be no sense in the ratio of the marginal of X to that of Y, if one set of quantities is possessed when the marginal utility of X is calculated, and another set when we calculate the marginal utility of Y.

In order to avoid the danger of misleading associations, let us give this quantity a new name, and call it the Marginal Rate of Substitution between the two commodities. We may define the marginal rate of substitution of X for Y as the quantity of Y which would just compensate the consumer for the loss of a marginal unit of X. This definition is entirely free from any dependence upon a quantitative measure of utility.

If an individual is to be in equilibrium with respect to a system of market prices, it is directly evident that his marginal rate of substitution between any two goods must equal the ratio of their prices. Otherwise he would clearly find an advantage in substituting some quantity of one for an equal value (at the market rate) of the other. This is therefore the form in which we must now write the condition of equilibrium on the market.

It may be observed that in this formulation we have, as yet, scarcely departed from Marshall. The marginal rate of substitution of X for Y is what he would have called the marginal utility of X in terms of Y. We may transcribe Marshall if we like, and say that the price of a commodity equals the marginal rate of substitution of that commodity for money.

7. The second victim (a more serious one this time) must be the principle of Diminishing Marginal Utility. If marginal utility has no exact sense, diminishing marginal utility can have no exact sense either. But by what shall we replace it?

By the rule that the indifference curves must be convex to the axes. This may be called, in our present terminology, the principle of Diminishing Marginal Rate of Substitution.* It may be expressed in the following terms: Suppose we start with a given quantity of goods, and then go on increasing the amount of X and diminishing that of Y in such a way that the consumer is left neither better off nor worse off on balance; then the amount of Y which has to be subtracted in order to set off a second unit of X will be less than that which has to be subtracted in order to set off the first unit. In other words, the more X is substituted for Y, the less will be the marginal rate of substitution of X for Y.

* I must here apologize to the reader for a tiresome change in terminology. In 'A Reconsideration' I looked at the change the other way up, and therefore talked about an Increasing Marginal Rate of Substitution where I here talk about a diminishing rate. It will be obvious why this seemed at first sight more convenient. But I have now come to think that the advantage of keeping my terminology as close as possible to the familiar Marshall terminology outweighs this alight difference in convenience.



But what is the exact reason why we must replace diminishing marginal utility by precisely this principle-the principle of diminishing marginal rate of substitution? As we have seen already,* they are not exactly the same thing. The replacement is therefore not a mere translation; it is a positive change in the foundation of the theory, and requires a very definite justification.

* See above, p. 16.

The justification is this. We need the principle of diminishing marginal rate of substitution for the same reason as Marshall's theory needed the principle of diminishing marginal utility. Unless, at the point of equilibrium, the marginal rate of substitution is diminishing, equilibrium will not be stable. Even if the marginal rate of substitution equals the price ratio, so that the acquisition of one unit of X would not yield any appreciable advantage; nevertheless, if the marginal rate of substitution is increasing, the acquisition of a larger quantity would be advantageous. It is instructive to set this out on the indifference diagram (Fig. 4).

At the point Q on the diagram, the marginal rate of substitution equals the price-ratio, so that the price-line touches the indifference curve through Q. But the marginal rate of substitution is increasing (the indifference curve is concave to the axes), so that a movement away from Q in either direction along LM would lead the individual on to a higher indifference curve. Q is therefore a point of minimum, not maximum, utility, and cannot be a point of equilibrium.

It is clear, therefore, that for any point to be a possible rate of equilibrium at appropriate prices the marginal rate of substitution at that point must be diminishing. Since we know from experience that some points of possible equilibrium do exist on the indifference maps of nearly every one (that is to say, they do decide to buy such-and-such quantities of commodities, and do not stay hesitating indefinitely like Buridan's ass), it follows that the principle of diminishing marginal rate of substitution must sometimes be true.

However, for us to make progress in economics, it is not enough for us that the principle should be true sometimes; we require a more general validity than that. The law of diminishing marginal utility used to be assumed generally valid (with perhaps some special exceptions), and on that general validity important economic conclusions were based. We shall have to investigate those conclusions afresh; but, if they are to have any chance at all, they need as their basis a property of the indifference map which is more than *sometimes* true.

What were in fact the grounds upon which economists used to base their general principle of diminishing marginal utility? Usually an appeal to experience; though to experience of that uncomfortably vague sort which does not offer any opportunity for actual testing. Critics have not been lacking to point out that this procedure was not very scientific, and the doubts which have been thrown by our present discussion upon the intelligibility of the `law of diminishing marginal utility' itself can only strengthen the case against the traditional procedure. If, however, we throw over diminishing marginal utility as being in any case dubious, and now certainly irrelevant, can we base upon similar `experience' a general principle of diminishing marginal rate of substitution? Again, I suppose, we might get away without being challenged; but one would like a surer foundation.

8. We can, I think, get that surer foundation if we reflect on the purpose for which we require our principle. We want to deduce from it laws of market conduct --laws, that is, which deal with the reaction of the consumer to changes in market conditions. When market conditions change, the consumer moves from one point of equilibrium to another point of equilibrium; at each of these positions the condition of diminishing marginal rate of substitution must hold, or he could not take up such a position at all. So much is clear directly; but to proceed from this to the law of diminishing marginal rate of substitution, as we need it in economic theory, an assumption is necessary. We have to assume that the condition holds at all intermediate points, so that there are no kinks in the curves between the two positions of equilibrium. (If there are kinks in the curves, curious consequences follow, such that there will be some systems of prices at which the consumer will be unable to choose between two different ways of spending his income.) The general principle of diminishing marginal rate of substitution merely rules out these oddities; by that principle we select the simplest of the various possibilities before us.

As we go on, we shall find that most of the 'laws' of pure economic theory can be looked at in this sort of way. Pure economics has a remarkable way of producing rabbits out of a hat --apparently *a priori* propositions which apparently refer to reality. It is fascinating to try to discover how the rabbits got in; for those of us who do not believe in magic must be convinced that they got in somehow. I have become convinced myself that they get in in two ways. One is by the assumption, at the beginning of every economic argument, that the things to be dealt with in the argument are the only things that matter in some practical problem. (This is always a dangerous assumption, and nearly always more or less wrong-which is why the application of economic theory is such a ticklish matter.) That takes us much of the way, but it does not take us the whole way. The other assumption is that which we have just isolated, an assumption that kinks can be neglected that there is a sufficient degree of regularity in the system of wants also, as we shall see later, in the productive system) for any set of quantities in the neighbourhood of those with which we are concerned to be a possible position of equilibrium at some system of prices. Again, this assumption may be wrong; but, being the simplest assumption possible, it is a good assumption tin to start with; and in fact its accordance with experience seems definitely good.

The road which lies before us now begins to be distinguishable. If this is the true foundation of the principle of diminishing marginal rate of substitution among consumption goods, other principles can be discovered whose foundation is exactly similar. These principles can be enumerated, and their consequences worked out. Some of them deal with production, and will be considered in Chapter VI below; the rest are extensions, into one field or another, of the principle elicited in this chapter. That there are a great many such extensions appears at once when we consider how wide is the variety of human choices which can be fitted into the framework of the Paretian scale of preference. What begins as an analysis of the consumer's choice among consumption goods ends as a theory of economic choice in general. We are in sight of a unifying principle for the whole of economics.

9. But this is running ahead. Before we can explore these long avenues much preparation is needed. One necessary piece of preparation may conclude this chapter.

During most of the above discussion we have made the extreme simplification that

the consumer had his choice restricted to expenditure on two sorts of goods. It is high time that we abandoned this simplification, for if our theory were confined to this simple case there would not be much to be said for it. It is in fact one of the main defects of the indifference-curve technique that it encourages concentration upon this simple case, concentration that can easily prove very dangerous.

When expenditure is distributed between more than two goods, the indifference diagram loses its simplicity; for three goods we need three dimensions, and for more than three goods geometry fails us altogether. However, the principles which we have established in this chapter remain substantially unaffected. The marginal rate of substitution can be defined as before, with the added proviso that the quantities consumed of all other commodities (Z...) must remain unchanged. The consumer is only in full equilibrium if the marginal rate of substitution between any two goods equals their price-ratio. Over the principle of diminishing marginal rate of substitution there is a slight difference.

In order that equilibrium should be stable, when expenditure is distributed among many commodities, it is necessary that no possible substitution of equal market values should lead the consumer to a preferred position. This means not only that we must have a diminishing marginal rate of substitution between each pair of commodities, but also that more complicated substitutions (of some X for some Y and some Z) must be ruled out in the same way. We may express this by saying that the marginal rate of substitutions must diminish for substitutions in every direction. This is a rather complicated condition, but it will appear, as we proceed, that it leads directly to conclusions of great importance.

On the same grounds as before, we shall assume that the marginal rate of substitution diminishes in every direction at every position with which we shall be concerned in our analysis. I do not think this could be established introspectively, or from 'experience', but it can be justified in the same way as we have justified the simpler condition. It becomes clear now, however, that it is a fairly drastic hypothesis, which gives us a good deal to go on, and from which we can expect to deduce some positive results.