Space for Notes
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Mathematical Principles of the Theory of Wealth

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Chapter IV

Of the Law of Demand

20. To lay the foundations of the theory of exchangeable values, we shall not accompany most speculative writers back to the cradle of the human race; we shall undertake to explain neither the origin of property nor that of exchange or division of labour. All this doubtless belongs to the history of mankind, but it has no influence on a theory which could only become applicable at a very advanced state of civilization, at a period when (to use the language of mathematicians) the influence of the *initial* conditions is entirely gone.

We shall invoke but a single axiom, or, if you prefer, make but a single hypothesis, i.e. that each one seeks to derive the greatest possible value from his goods or his labour. But to deduce the rational consequences of this principle, we shall endeavour to establish better than has been the case the elements of the data which observation alone can furnish. Unfortunately, this fundamental point is one which theorists, almost with one accord, have presented to us, we will not say falsely, but in a manner which is really meaningless.

It has been said almost unanimously that "the price of goods is in the inverse ratio of the quantity offered, and in the direct ratio of the quantity demanded." It has never been considered that the statistics necessary for accurate numerical estimation might be lacking, whether of the quantity offered or of the quantity demanded, and that this might prevent deducing from this principle general consequences capable of useful application. But wherein does the principle itself consist? Does it mean that in case a double quantity of any article is offered for sale, the price will fall one-half? Then it should be more simply expressed, and it should only be said that the price is in the inverse ratio of the quantity offered. But the principle thus made intelligible would be false; for, in general, that 100 units of an article have been sold at 20 francs is no reason that zoo units would sell at 10 francs in the same lapse of time and under the same circumstances. Sometimes less would be marketed; often much more.

Furthermore, what is meant by the quantity demanded? Undoubtedly it is not that which is actually marketed at the demand of buyers, for then the generally absurd consequence would result from the pretended principle, that the more of an article is marketed the dearer it is. If by demand only a vague desire of possession of the article is understood, without reference to the *limited price* which every buyer supposes in his demand, there is scarcely an article for which the demand cannot be considered indefinite; but if the price is to be considered at which each buyer is willing to buy, and the price at which each seller is willing to sell, what becomes of the pretended principle? It is not, we repeat, an erroneous proposition - it is a proposition devoid of meaning. Consequently all those who have united to proclaim it have likewise united to make no use of it. Let us try to adhere to less sterile principles.

The cheaper an article is, the greater ordinarily is the demand for it. The sales or the demand (for to us these two words are synonymous, and we do not see for what reason theory need take account of any demand which does not result in a sale)-the sales or the demand generally, we say, increases when the price decreases.

We add the word generally as a corrective; there are, in fact, some objects of whim and luxury which are only desirable on account of their rarity and of the high price which is the consequence thereof. If any one should succeed in carrying out cheaply the crystallization of carbon, and in producing for one franc the diamond which to-day is worth a thousand, it would not be astonishing if diamonds should cease to be used in sets off jewellery, and should disappear as articles of commerce. In this case a great fall in price would almost annihilate the demand. But objects of this nature play so unimportant a part in social economy that it is not necessary to bear in mind the restriction of which we speak.

The demand might be in the inverse ratio of the price; ordinarily it increases or decreases in much more rapid proportion - an observation especially applicable to most manufactured products. On the contrary, at other times the variation of the demand is less rapid; which appears (a very singular thing) to be equally applicable both to the most necessary things and to the most superfluous. The price of violins or of astronomical telescopes might fall one-half and yet probably the demand would not double; for this demand is fixed by the number of those who cultivate the art or science to which these instruments belong; who have the disposition requisite and the leisure to cultivate them and the means to pay teachers and to meet the other necessary expenses, in consequence of which the price of the instruments is only a secondary question. On the contrary, firewood, which is one of the most useful articles, could probably double in price, from the progress of clearing land or increase in population, long before the annual consumption of fuel would be halved; as a large number of consumers are disposed to cut down other expenses rather than get along without firewood.

21. Let us admit therefore that the sales or the annual demand ***D*** is, for each article, a particular function ***F(p)*** of the price ***p*** of such article. To know the form of this function would be to know what we call *the law of demand* or of sales. It depends evidently on the kind of utility of the article, on the nature of the services it can render or the enjoyments it can procure, on the habits and customs of the people, on the average wealth, and on the scale on which wealth is distributed.

Since so many moral causes capable of neither enumeration nor measurement affect the law of demand, it is plain that we should no more expect this law to be expressible by an algebraic formula than the law of mortality, and all the laws whose determination enters into the field of statistics, or what is called social arithmetic. Observation must therefore be depended on for furnishing the means of drawing up between proper limited a table of the corresponding values of ***D*** and ***p***; after which, by the well-known methods of interpolation or by graphic processes, an empiric formula or a curve can be made to represent the function in question; and the solution of problems can be pushed as far as numerical applications.

But even if this object were unattainable (on account of the difficulty of obtaining observations of sufficient number and accuracy, and also on account of the progressive variations which the law of demand must undergo in a country which has not yet reached a practically stationary condition), it would be nevertheless not improper to introduce the unknown law of demand into analytical combinations, by means of an indeterminate symbol; for it is well known that one of the most important functions of analysis consists precisely in assigning determinate relations between quantities to which numerical values and even algebraic forms are absolutely unassignable.

Unknown functions may nonetheless possess properties or general characteristics which are known; as, for instance, to be indefinitely increasing or decreasing, or periodical, or only real between certain limits. Nevertheless such data, however imperfect they may seem, by reason of their very generality and by means of analytical symbols, may lead up to relations equally general which would have been difficult to discover without this help. Thus without knowing the law of decrease of the capillary forces, and starting solely from the principle that these forces are inappreciable at appreciable distances, mathematicians have demonstrated the general laws of the phenomena of capillarity, and these laws have been confirmed by observation.

On the other hand, by showing what determinate relations exist between unknown quantities, analysis reduces these unknown quantities to the smallest possible number, and guides the observer to the best observations for discovering their values. It reduces and coordinates statistical documents; and it diminishes the labour of statisticians at the same time that it throws light on them.

For instance, it is impossible a priori to assign an algebraic form to the law of mortality; it is equally impossible to formulate the function expressing the subdivision of population by ages in a stationary population; but these two functions are connected by so simple a relation, that, as soon as statistics have permitted the construction of a table of mortality, it will be possible, without recourse to new observations, to deduce from this table one expressing the proportion of the various ages in the midst of a stationary population, or even of a population for which the annual excess of deaths over births is known.*

* The *Annuaire du Bureau des Longitudes* contains these two tables, the second deduced from the first, as above, and calculated on the hypothesis of a stationary population.
The work by Duvillard, entitled *De l"influence de la petite virole sur la mortalite* contains many good examples of mathematical connections between essentially empirical functions.

Who doubts that in the field of social economy there is a mass of figures thus mutually connected by assignable relations, by means of which the easiest to determine empirically might be chosen, so as to deduce all the others from it by means of theory?

22. We will assume that the function F(p), which expresses the law of demand or of the market, is a *continuous* function, i.e. a function which does not pass suddenly from one value to another, but which takes in passing all intermediate values. It might be otherwise if the number of consumers were very limited: thus in a certain household the same quantity of firewood will possibly be used whether wood costs 10 francs or 15 francs the stem and the consumption may suddenly be diminished if the price of the *stere** rises above the latter figure. But the wider the market extends, and the more the combinations of needs, of fortunes, or even of caprices, are varied among consumers, the closer the function *F(p)* will come to varying with pin a continuous manner. However little may be the variation of p, there will be some consumers so placed that the slight rise or fall of the article will affect their consumptions, and will lead them to deprive themselves in some way or to reduce their manufacturing

output, or to substitute something else for the article that has grown dearer, as, for instance, coal for wood or anthracite for soft coal. Thus the " exchange " is a thermometer which shows by very slight variations of rates the fleeting variations in the estimate of the chances which affect government bonds, variations which are not a sufficient motive for buying or selling to most of those who have their fortunes invested in such bonds.

[* 1 *stere* = 1 M^3 = 35.3 cu. ft. = 1% cord.-TRANSLATOR.]

If the function ***F(p)*** is continuous, it will have the property common to all functions of this nature, and on which so many important applications of mathematical analysis are based : the *variations of the demand wall be sensibly proportional to the variations in price so long as these last are small fractions of the original price.* Moreover, these variations will be of opposite signs, *i.e.* an increase in price will correspond with a diminution of the demand.

Suppose that in a country like France the consumption of sugar is 100 million kilograms when the price is 2 francs a kilogram, and that it has been observed to drop to 99 millions when the price reached 2 francs 10 centimes. Without considerable error, the consumption which would correspond to a price of 2 francs 20 centimes can be valued at 98 millions, and the consumption corresponding to a price of 1 franc go centimes at 101 millions. It is plain how much this principle, which is only the mathematical consequence of the continuity of functions, can facilitate applications of theory, either by simplifying analytical expressions of the laws which govern the movement of values, or in reducing the number of data to be borrowed from experience, if the theory becomes sufficiently developed to lend itself to numerical determinations.

Let us not forget that, strictly speaking, the principle just enunciated admits of exceptions, because a continuous function may have interruptions of continuity in some points of its course ; but just as friction wears down roughnesses and softens outlines, so the wear of commerce tends to suppress these exceptional cases, at the same time that commercial machinery moderates variations in prices and tends to maintain them between limits which facilitate the application of theory.

23. To define with accuracy the quantity ***D***, or the function ***F(p)*** which is the expression of it, we have supposed that ***D*** represented the quantity sold *annually* throughout the extent of the country or of the market* under consideration. In fact, the year is the natural unit of time, especially for researches having any connection with social economy. All the wants of mankind are reproduced during this terra, and all the resources which mankind obtains from nature and by labour. Nevertheless, the price of an article may vary notably in the course of a year, and, strictly speaking, the law of demand may also vary in the same interval, if the country experiences a movement of progress or decadence. For greater accuracy, therefore, in the expression ***F(p), p*** must be held to denote the annual average price, and the curve which represents function ***F*** to be in itself an average of all the curves which would represent this function at different times of the year. But this extreme accuracy is only necessary in case it is proposed to go on to numerical applications, and it is superfluous for researches which only seek to obtain a general expression of average results, independent of periodical oscillations.

* It is well known that by market economists mean, not a certain place where purchases and sales are carried on, but the entire territory of which the parts are so united by the relations of

unrestricted commerce that prices there take the same level throughout, with ease and rapidity.

24. Since the function ***F(p)*** is continuous, the function ***pF(p),*** which expresses the total value of the quantity annually sold, must be continuous also. This function would equal zero if ***p*** equals zero, since the consumption of any article remains finite even on the hypothesis that it is absolutely free; or, in other words, it is theoretically always possible to assign to the symbol ***p*** a value so small that the product ***pF(p)*** will vary imperceptibly from zero. The function ***pF(p)*** disappears also when ***p*** becomes infinite, or, in other words, theoretically a value can always be assigned to ***p*** so great that the demand for the article and the production of it would cease. Since the function ***pF(p)*** at first increases, and then decreases as ***p*** increases, there is therefore a value of ***p*** which makes this function a maximum, and which is given by the equation,

$$F(p) + pF'(p) = 0, \qquad \text{(I)}$$

in which ***F'***, according to Lagrange's notation, denotes the differential coefficient of function ***F.***

If we lay out the curve ***anb*** (Fig. I), of which the abscissas ***oq*** and the ordinates ***qn*** represent the variables ***p*** and ***D***, the root of equation (I) will be the abscissa of the point ***n*** from which the triangle ***ont*** formed by the tangent ***nt*** and the radius vector ***on***, is isosceles, so that we have ***oq = qt***.

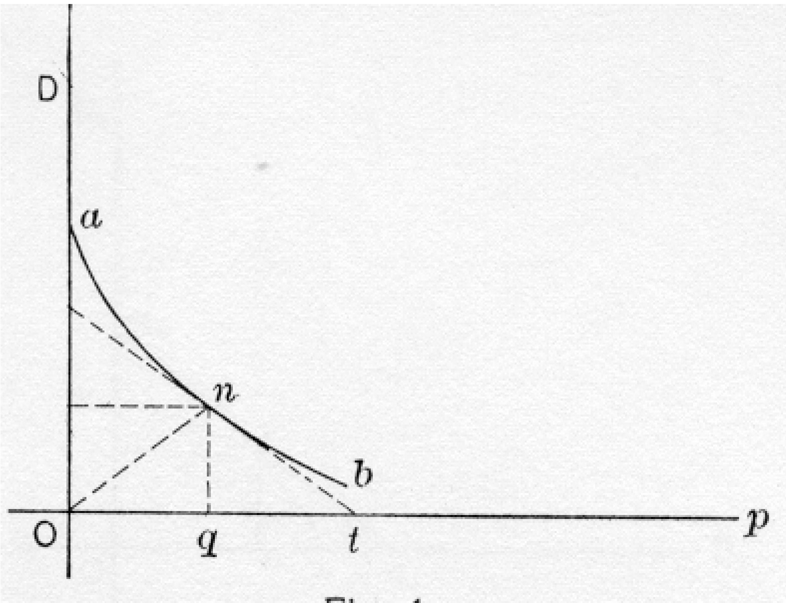

Fig. 1.

We may admit that it is impossible to determine the function ***F(p)*** empirically for each article, but it is by no means the case that the same obstacles prevent the approximate determination of the value of ***p*** which satisfies equation (I) or which renders the product ***pF(p)*** a maximum. The construction of a table, where these values could be found, would be the work best calculated for preparing for the practical and rigorous solution of questions relating to the theory of wealth.

But even if it were impossible to obtain from statistics the value of ***p*** which should render the product ***pF(p)*** a maximum, it would be easy to learn, at least for all articles to which the attempt has been made to extend commercial statistics, whether current prices are above or below this value. Suppose that when the price becomes $p + \Delta p$, the annual consumption as shown by statistics, such as customhouse records, becomes $D - \Delta D$. According as

$$\Delta D/\Delta p < \text{or} > D/p,$$

the increase in price, Δp, will increase or diminish the product $pF(p)$; and, consequently, it will be known whether the two values p and $p + \Delta p$ (assuming Δp to be a small fraction of p) fall above or below the value which makes the product under consideration a maximum.

Commercial statistics should therefore be required to separate articles of high economic importance into two categories, according as their current prices are above or below the value which makes a maximum of $pF(p)$**.** We shall see that many economic problems have different solutions, according as the article in question belongs to one or the other of these two categories.

25. We know by the theory of maxima and minima that equation (I) is satisfied as well by the values of p which render $pF(p)$ a minimum as by those which render this product a maximum. The argument used at the beginning of the preceding article shows, indeed, that the function $pF(p)$ necessarily has a maximum, but it might have several and pass through minimum values between. A root of equation (I) corresponds to a maximum or a minimum according as

$$2\,F'(p) + pF''(p) < \text{or} > 0,$$

or, substituting for p its value and considering the essentially negative sign of $F'(p)$,

$$2\,[F'(p)]^2 - F(p) \times F''(p) > \text{or} < 0.$$

In consequence, whenever $F''(p)$ is negative, or when the curve $D = F(p)$ turns its concave side to the axis of the abscissas,, it is impossible that there should be a minimum, nor more than one maximum. In the contrary case, the existence of several maxima or minima is not proved to be impossible.

But if we cease considering the question from an exclusively abstract standpoint, it will be instantly recognized how improbable it is that the function $pF(p)$ should pass through several intermediate maxima and minima inside of the limits between which the value of p can vary; and as it is unnecessary to consider maxima which fall beyond these limits, if any such exist, all problems are the same as if the function $pF(p)$ only admitted a single maximum. The essential question is always whether, for the extent of the limits of oscillation of p, the function $pF(p)$ is increasing or decreasing for increasing values of p.

Any demonstration ought to proceed from the simple to the complex: the simplest hypothesis for the purpose of investigating by what laws prices are fixed, is that of monopoly, taking this word in its most absolute meaning, which supposes that the production of an article is in one man's hands. This hypothesis is not purely fictitious : it is realized in certain cases; and, moreover, when we have studied it, we can analyze more accurately the effects of competition of producers.